

A.2 Forces

Ahmed Gomaa · IB DP Physics SL and HL · Part 1 of 3 in A.2 (next: circular motion, then linear momentum) · original worked examples

Contents

- | | |
|---|--------------------------------------|
| 1. <u>The big picture</u> | 11. <u>Newton's second law</u> |
| 2. <u>The forces you need</u> | 12. <u>Lifts and apparent weight</u> |
| 3. <u>Friction</u> | 13. <u>The inclined plane</u> |
| 4. <u>Buoyancy (upthrust)</u> | 14. <u>Connected bodies</u> |
| 5. <u>Viscous drag and Stokes' law</u> | 15. <u>Pulleys: the two set-ups</u> |
| 6. <u>Terminal speed in a fluid</u> | 16. <u>One block on another</u> |
| 7. <u>Free-body diagrams</u> | 17. <u>Writing the explanations</u> |
| 8. <u>Resultant force and equilibrium</u> | 18. <u>Common mistakes</u> |
| 9. <u>Newton's first law</u> | 19. <u>Practise next</u> |
| 10. <u>Newton's third law</u> | |

Pen colours: blue = bodies and diagrams, red = forces, laws and key steps, green = answers and accelerations, pencil = construction lines and side notes. Boxes: **Why?** = the reasoning, **Mistake** = what students get wrong, yellow note = exam tip. Use $g = 9.8 \text{ m s}^{-2}$ unless told otherwise.

The big picture

In A.1 we **described** motion. Now we ask **why** motion changes. The answer is always the same: a **net force**.

A **force** is an interaction between two bodies: a push or a pull.
Force is a **vector**: it has a size (in newtons, N) and a direction.

The whole topic rests on a few big ideas:

1. **Identify the forces** on a body and draw them in a free-body diagram.
2. **Add them** to find the net (resultant) force.
3. **Use Newton's laws:** zero net force $\rightarrow$ constant velocity; net force $\rightarrow$ acceleration $F = ma$.
4. Later in A.2 the same laws explain **circular motion** and **momentum** (separate notes).

Why "interaction"?

A force always involves **two** bodies: the Earth pulls you, the floor pushes your feet, a string pulls a box. If you cannot name the second body, the force does not exist. This one check removes most wrong forces from diagrams, such as "the force of motion".

The forces you need

In this topic you meet two families of forces. **Field forces** act at a distance: gravitational, electric and magnetic. **Contact forces** need touching: normal force, friction, tension, the spring force, drag and buoyancy.

Weight (gravitational force)

$$F_g = mg \quad \text{always vertically down, towards the centre of the planet}$$

Mass (kg) is the amount of matter and does not change from place to place. **Weight** (N) is the pull of the planet, so it depends on g . A 70 kg astronaut has a weight of $70 \times 9.8 = 686$ N on Earth but only $70 \times 1.6 = 112$ N on the Moon. Her mass is 70 kg in both places. The unit of g , N kg^{-1} , is the same as m s^{-2} .

Tension

A taut string, rope or cable pulls on whatever is attached to each end. The tension always acts **along the string, away from the body** (a string can

pull, never push). We treat strings as **light** (massless) and **inextensible**: then the tension is the same all along the string, and the bodies joined by it have the same speed and the same size of acceleration. A slack string has zero tension.

The spring force and Hooke's law

A spring that is stretched or squashed pushes or pulls to get back to its natural length. For many springs the force is proportional to the extension x :

$$F_H = -kx \quad k = \text{spring constant (N m}^{-1}\text{)}$$

The minus sign: the force is opposite to the extension (a **restoring force**).

A stiff spring has a large k . For example, a 2.0 kg mass hangs at rest from a spring with $k = 250 \text{ N m}^{-1}$. The spring force balances the weight: $kx = mg$, so $x = (2.0 \times 9.8)/250 = 0.078 \text{ m} = 7.8 \text{ cm}$. On the Moon the extension would be smaller, because the weight is smaller.

Normal force

When two surfaces touch, each pushes the other **at right angles (normal) to the surface**. The normal force is the part (component) of the contact force that is perpendicular to the surface. The part parallel to the surface is friction. The atoms in the surface are squashed a tiny amount and push back, like many small springs. The normal force F_N adjusts itself to stop the body sinking into the surface.

✗ Mistake

"The normal force always equals mg ." Only for a body resting on a horizontal surface with no other vertical forces. Push down on a book and F_N grows; put the book on a slope and $F_N = mg \cos \theta$; in a lift that accelerates, F_N changes again. Always find F_N from the forces, never assume it.

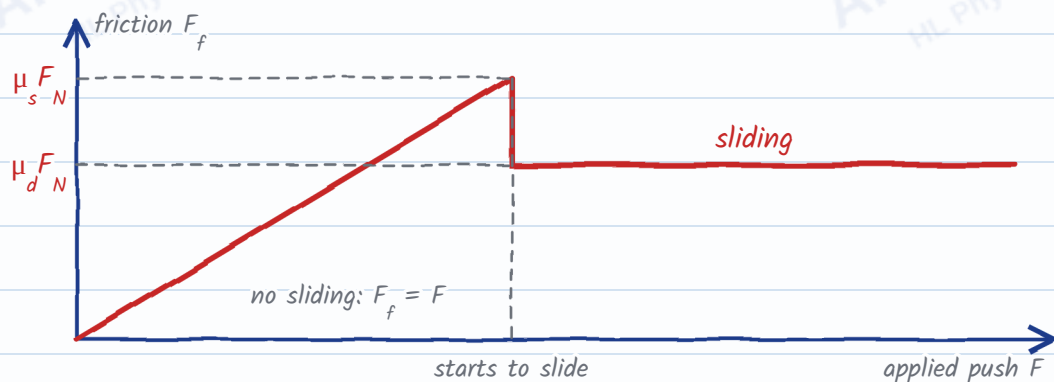
Friction

Friction acts **along** the surface, opposing sliding or the **tendency** to slide. It comes from the tiny bumps of the two surfaces catching on each other and from forces between their atoms.

Static (no sliding): $F_f \leq \mu_s F_N$

Dynamic (sliding): $F_f = \mu_d F_N$ with $\mu_d < \mu_s$ (usually)

Static friction is "as much as needed, up to a limit". Push gently on a heavy box: it does not move, so friction exactly matches your push. Push harder and friction grows with you, until it reaches its maximum $\mu_s F_N$. Push beyond that and the box slides; friction then drops to the smaller dynamic value $\mu_d F_N$, which stays about the same at any sliding speed.



Static friction matches the push until it reaches $\mu_s F_N$; once sliding starts, friction is $\mu_d F_N$.

Worked example 1

A 40 kg crate rests on a level floor. $\mu_s = 0.50$ and $\mu_d = 0.35$. Find the friction force when a worker pushes horizontally with (a) 150 N, (b) 250 N. In (b) find the acceleration.

1. $F_N = mg = 40 \times 9.8 = 392$ N. Maximum static friction = $0.50 \times 392 = 196$ N.

2. (a) $150\text{ N} < 196\text{ N}$, so the crate stays at rest. Friction balances the push: $F_f = 150\text{ N}$ (not 196 N).
3. (b) $250\text{ N} > 196\text{ N}$, so it slides. $F_f = \mu_d F_N = 0.35 \times 392 = 137\text{ N}$.
4. $a = (250 - 137)/40 = 2.8\text{ m s}^{-2}$

✗ Mistake

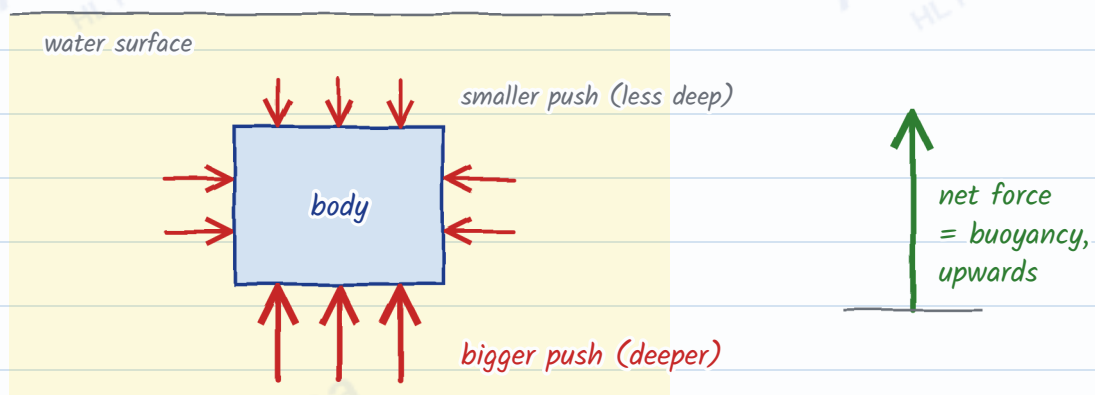
Writing $F_f = \mu_s F_N$ for a body at rest. That formula gives the **largest possible** static friction, which only acts when the body is just about to slip. Notice also that friction does not depend on the area of contact or (for sliding) on the speed.

Buoyancy (upthrust)

Hold a football under water and you feel it push up hard. Every body in a fluid (a liquid or a gas) feels an upward force called **buoyancy** or upthrust, F_b .

Where does it come from?

The pressure in a fluid increases with depth, because the deeper you go, the more fluid is resting on top. So the bottom of a submerged body is pushed up harder than its top is pushed down. The forces on the sides cancel. What is left is a net **upward** force.



The pressure is larger at the bottom, so the fluid pushes up more than it pushes down.

A neat argument gives the size of the force. Imagine the body is removed and the hole is filled with the same fluid. That "fluid body" would float at rest, so the upward push of the surrounding fluid must equal its weight. The real body has exactly the same surroundings, so it feels the same push. This is **Archimedes' principle**:

Buoyancy = weight of the fluid displaced

$F_b = \rho V g$ ρ = density of the fluid, V = volume of fluid displaced

(the part of the body that is under the surface)

✗ Mistake

Using the density of the **body** in $F_b = \rho V g$. The buoyancy depends only on the fluid and the volume pushed aside. A steel ball and a plastic ball of the same size, fully under water, feel exactly the same buoyancy.

Floating and sinking

Fully submerged, the body's weight is $\rho_{\text{body}} V g$ and the buoyancy is $\rho_{\text{fluid}} V g$.

So the comparison is just a comparison of densities:

Densities

What happens

$\rho_{\text{body}} > \rho_{\text{fluid}}$ weight > buoyancy: the body sinks (it accelerates down, then may reach a terminal speed)

$\rho_{\text{body}} = \rho_{\text{fluid}}$ forces balance at any depth: the body stays where it is put

$\rho_{\text{body}} < \rho_{\text{fluid}}$ buoyancy > weight: it rises and floats with part of it above the surface

A floating body sinks just deep enough for the buoyancy to equal its weight:

$\rho_{\text{fluid}} V_{\text{under}} g = \rho_{\text{body}} V_{\text{total}} g$, so

floating: $\text{fraction under the surface} = V_{\text{under}} / V_{\text{total}} = \rho_{\text{body}} / \rho_{\text{fluid}}$

Worked example 2

A wooden block of density 650 kg m^{-3} and volume $2.0 \times 10^{-3} \text{ m}^3$ floats in water (1000 kg m^{-3}). Find (a) the fraction under water, (b) the buoyancy, (c) the extra downward push needed to hold it completely under water.

1. (a) fraction = $650/1000 = 0.65$: 65 % is under water.
2. (b) Floating, so buoyancy = weight = $\rho_{\text{body}} V g = 650 \times 2.0 \times 10^{-3} \times 9.8 = 12.7 \text{ N}$.
3. (c) Fully under: $F_b = 1000 \times 2.0 \times 10^{-3} \times 9.8 = 19.6 \text{ N}$. Forces balance: push + weight = buoyancy $\rightarrow$ push = $19.6 - 12.7 = 6.9 \text{ N}$.

Worked example 3

A 2.7 kg aluminium block (density 2700 kg m^{-3}) hangs from a string, fully under water. Find the tension in the string. Compare it with the tension in air.

1. Volume $V = m/\rho = 2.7/2700 = 1.0 \times 10^{-3} \text{ m}^3$.
2. Three forces, in equilibrium: $T + F_b = mg$.
3. $F_b = 1000 \times 1.0 \times 10^{-3} \times 9.8 = 9.8 \text{ N}$; $mg = 2.7 \times 9.8 = 26.5 \text{ N}$.
4. $T = 26.5 - 9.8 = 16.7 \text{ N}$. In air (buoyancy of air is tiny) $T = 26.5 \text{ N}$. The block seems "lighter" in water: this is the **apparent weight**.

Why does a steel ship float?

The steel is much denser than water, but the hull is mostly air. What matters is the average density of the whole ship: its total mass divided by the volume of the hull below the deck. A ship floats lower when it is loaded, because it must displace more water to balance the bigger weight.

Viscous drag and Stokes' law

A body moving through a fluid feels a **drag force** that opposes its velocity relative to the fluid. In A.1 you met drag from air (a skydiver: at terminal speed the resultant force is zero, and in air the buoyancy is tiny, so drag $\approx$ weight). Now we look at a case we can calculate: a small sphere moving slowly through a **viscous** liquid, like a steel ball sinking in honey or oil.

Viscosity

Viscosity η measures how "thick" a fluid is: how strongly its layers resist sliding over each other. Honey and engine oil have a high viscosity; water and air have a low viscosity. For liquids, viscosity falls quickly as the temperature rises (warm honey pours easily).

$$\text{Stokes' law: } F_d = 6\pi\eta rv$$

η = viscosity (Pa s) · r = radius of the sphere · v = speed of the sphere through the fluid

Each factor makes sense: a thicker fluid (bigger η), a bigger sphere (bigger r) and a faster sphere (bigger v) all give more drag. The drag is **proportional to the speed**.

When can you use Stokes' law?

Only for a **small sphere moving slowly** through a fluid with **smooth (laminar) flow**, far from the walls of the container. It

does **not** apply to a skydiver, a car or a fast ball in air: there the flow is turbulent and the drag grows roughly with v^2 .

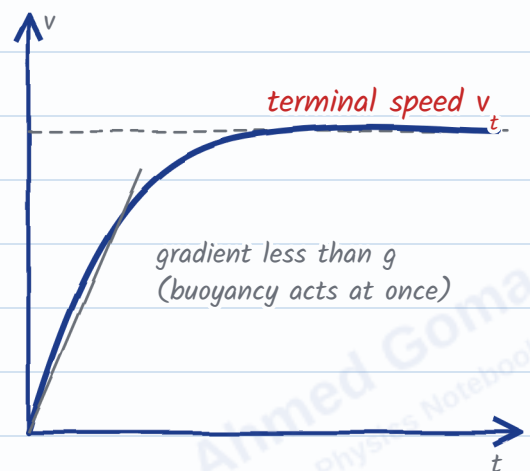
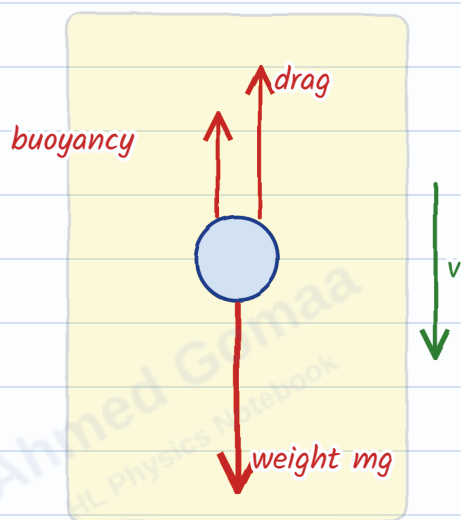
Where does the unit Pa s come from?

Rearrange: $\eta = F_d / (6\pi r v)$. Units: $\text{N} \div (\text{m} \times \text{m s}^{-1}) = \text{N s m}^{-2}$. Since $1 \text{ Pa} = 1 \text{ N m}^{-2}$, this is **Pa s**. Water has $\eta \approx 1 \times 10^{-3} \text{ Pa s}$; glycerine about 1.5 Pa s .

In **fundamental SI units** ($\text{N} = \text{kg m s}^{-2}$): η is measured in $\text{kg m s}^{-2} \div (\text{m} \times \text{m s}^{-1}) = \text{kg m}^{-1} \text{ s}^{-1}$. In the same way, if the air drag on a car is written $F = cv^2$, the constant c is measured in $\text{kg m s}^{-2} \div \text{m}^2 \text{ s}^{-2} = \text{kg m}^{-1}$.

Terminal speed in a fluid

Drop a small steel ball into a tall tube of glycerine. Three forces act on it:



Weight down; buoyancy and drag up (drag opposes the velocity).

The ball speeds up, the drag grows, the acceleration falls to zero.

At first $v = 0$, so there is no drag. The net force is $mg - F_b$ and the ball accelerates. As it speeds up, the drag $6\pi\eta r v$ grows, the net force gets

smaller and so does the acceleration. Soon the **resultant force is zero** and the ball falls at a constant **terminal speed** v_t .

At terminal speed the resultant force is zero: **weight = buoyancy + drag**

$$\rho_s V g = \rho_f V g + 6\pi\eta r v_t \quad \text{with } V = \frac{4}{3} \pi r^3$$

Rearranging (divide everything by $6\pi\eta r$):

$$v_t = \frac{2r^2 g (\rho_s - \rho_f)}{9\eta}$$

Derive, do not memorise

This formula is not in the data booklet. Exams ask you to **show** it from the force balance. Write the three forces, set up = down, put in $V = \frac{4}{3} \pi r^3$, and divide.

Worked example 4

A steel ball of radius 1.5 mm (density 7800 kg m^{-3}) falls through glycerine (density 1260 kg m^{-3} , viscosity 1.5 Pa s). Find its terminal speed. What answer would you get if you forgot the buoyancy?

1. $v_t = \frac{2r^2 g (\rho_s - \rho_f)}{9\eta} = \frac{2 \times (1.5 \times 10^{-3})^2 \times 9.8 \times (7800 - 1260)}{9 \times 1.5}$
2. = **0.021 m s⁻¹** (about 2 cm every second)
3. Without buoyancy (using 7800 instead of 6540): 0.025 m s^{-1} , about 19 % too big.

Why do big balls fall faster?

The weight grows with r^3 but the drag grows with r (at a given speed). So $v_t \propto r^2$: double the radius and the terminal speed becomes four times larger. This is also why tiny droplets in a cloud hardly fall at all.

A classic experiment

Measuring v_t for balls of different radius gives a way to find the viscosity of a liquid: plot v_t against r^2 ; the gradient is $2g(\rho_s - \rho_f)/9\eta$. Time the ball between two marks well below the surface, so it has already reached terminal speed. This makes a good IA or Paper 1B context.

Acceleration against speed: a straight line

Write $\Sigma F = ma$ for the sinking sphere: $ma = mg - F_b - 6\pi\eta rv$.

The drag is proportional to v , so a graph of **a** against **v** is a **straight line** with a negative gradient $(-6\pi\eta r/m)$. It starts at $a = (1 - \rho_f/\rho_s)g$ when $v = 0$ and reaches $a = 0$ at $v = v_t$.

Against **time**, the same motion gives a **curve**: a falls quickly at first and then more and more slowly towards zero.

Bodies that rise through a fluid

A body that is **less dense** than the fluid rises: a plastic bead released at the bottom of a tube of glycerine, an air bubble in water, a helium balloon. Just after release $v = 0$, so there is no drag. The net force is $F_b - W$, upwards.

Just after release: $a = (F_b - W)/m = (\rho_f - \rho_s)g/\rho_s$,
upwards

As the body speeds up, the drag grows. Drag always opposes the velocity, so on a rising body it points **down**. At terminal speed the resultant force is again zero:

Rising at terminal speed: $\text{buoyancy} = \text{weight} + \text{drag}$

Fully under the fluid, the weight and the buoyancy use the same volume, so $W/F_b = \rho_{\text{body}}/\rho_{\text{fluid}}$.

✗ Mistake

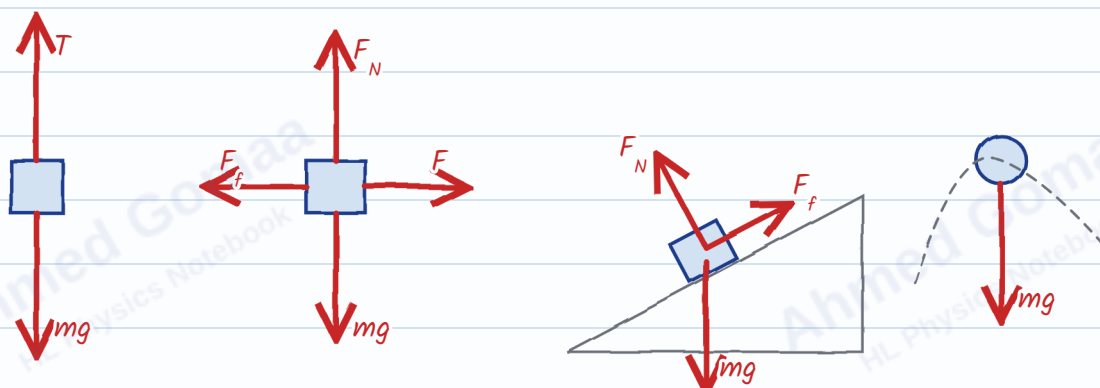
Drawing the drag upwards on a rising bubble or bead. Drag points against the velocity. On a body moving up it points down, so on the free-body diagram the buoyancy arrow must equal the weight and drag arrows together.

Free-body diagrams

A free-body diagram shows **one** body on its own, with every force that acts on it, drawn as an arrow starting from the body.

Rules for a full-mark diagram

1. One body only, drawn as a dot or a simple box.
2. Only forces acting **on** that body (not forces it exerts on others).
3. Each arrow starts at the body and points the way the force acts.
4. Lengths roughly to scale: equal forces get equal arrows.
5. Label each arrow with a standard name or symbol (W or mg , F_N , F_f , T , F_d , F_b).



A lamp at rest

B pushed, constant v

C at rest on a slope

D ball in flight

A: two equal forces. B: constant velocity, so push = friction. C: friction acts up the slope, against the tendency to slide. D: once the ball leaves the hand, no "throwing force" acts on it.

✗ Mistake

Adding forces that do not exist: "the force of motion", "the force of the throw" after the ball has left the hand, or a separate "centripetal force".

For every arrow, ask: **which other body exerts this force?** If there is no answer, delete the arrow.

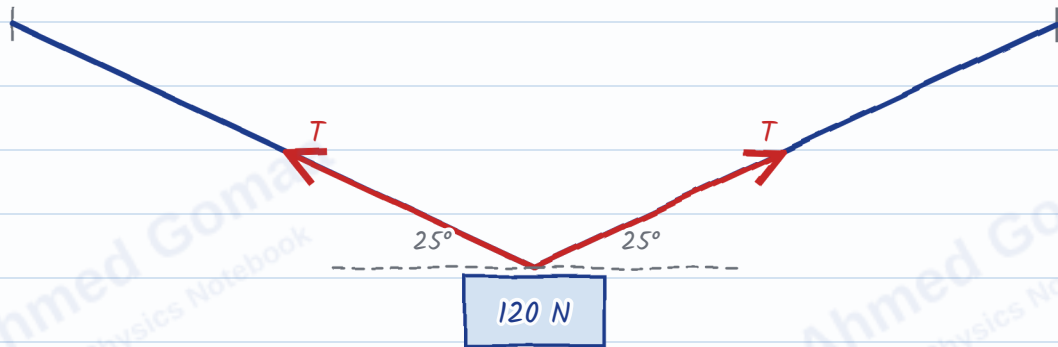
Resultant force and equilibrium

The **resultant** (net) force ΣF is the single force that has the same effect as all the forces together. Forces along one line just add with signs. For forces at angles, split each one into components along two perpendicular axes, add the components, and combine:

$$F_x = F \cos \theta \quad F_y = F \sin \theta \quad (\theta \text{ measured from the x-axis})$$

$$\Sigma F = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}, \text{ direction } \tan \phi = \Sigma F_y / \Sigma F_x$$

A body is in **translational equilibrium** when $\Sigma F = 0$. Then $\Sigma F_x = 0$ and $\Sigma F_y = 0$: two equations, so you can find two unknowns. If three forces are in equilibrium, their arrows drawn tip-to-tail form a closed triangle.



Each tension has a vertical component $T \sin 25^\circ$.

Worked example 5

A 120 N sign hangs from the middle of a cable. Each half of the cable makes 25° with the horizontal. Find the tension in the cable.

1. Horizontal: $T \cos 25^\circ$ (right) = $T \cos 25^\circ$ (left). The two tensions are equal by symmetry. ✓
2. Vertical: $2T \sin 25^\circ = 120$.
3. $T = 120 / (2 \times \sin 25^\circ) = 142 \text{ N}$

Why is the tension bigger than the weight?

Only a small part of each tension (the vertical component) holds the sign up. The flatter the cable, the smaller $\sin \theta$, and the larger T must be. A perfectly horizontal cable could never hold a weight: T would have to be infinite. This is why washing lines and power cables always sag.

Newton's first law

First law: if the net force on a body is zero, it stays at rest or keeps moving in a straight line at constant speed.

This says that motion does **not** need a force: a change in motion does. A puck on ice slides on almost forever; a car on the motorway needs its

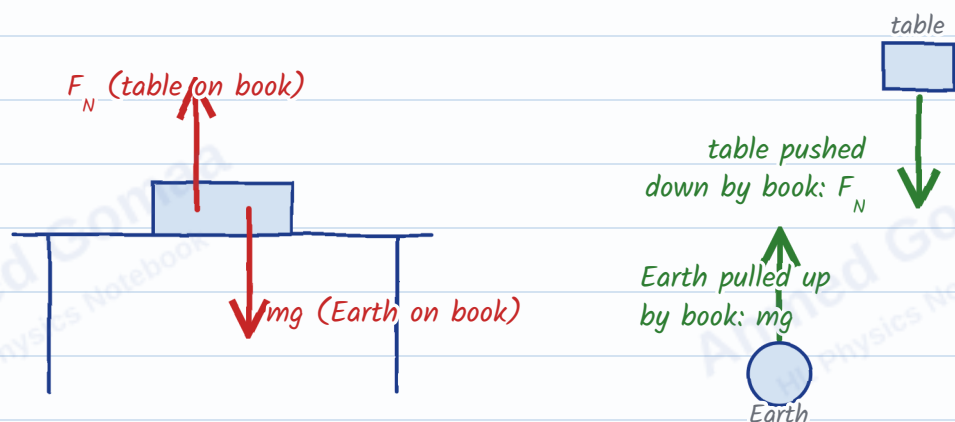
engine only because friction and air resistance would slow it down. At constant velocity, the forward force exactly balances the resistive forces. The tendency of a body to keep its velocity is called **inertia**, and mass is a measure of it. When a bus brakes suddenly, the passengers keep moving forward; when it pulls away, they seem to be pushed back into their seats. In both cases they are just keeping their original velocity.

Newton's third law

Third law: if body A exerts a force on body B, then B exerts a force on A that is equal in size and opposite in direction.

Forces always come in **pairs**. The two forces of a pair:

- act on **different** bodies (one on A, one on B), so they never cancel each other;
- are the **same type** of force (both gravitational, both contact, ...);
- act at the same time, for the same time.



Red: the two forces on the book. Green: their third-law partners, which act on other bodies.

Force

Earth pulls book down (weight)

table pushes book up (F_N)

Its third-law partner

book pulls Earth up (gravitational)

book pushes table down (contact)

rocket pushes exhaust gas back

gas pushes rocket forward

swimmer's hand pushes water back

water pushes swimmer forward

× Mistake

"The weight of the book and the normal force are a third-law pair." They are equal and opposite, but they act on the **same** body and are different types of force. They are equal only because the book is in equilibrium (first law). In an accelerating lift they are not equal at all.

Newton's second law

Second law: $\Sigma F = ma$

The net force and the acceleration are in the **same direction**.

(Constant mass only; the general form $\Sigma F = \Delta p / \Delta t$ is in the linear momentum notes.)

A free-body diagram tells you the direction of the **acceleration** (the direction of the resultant force). It cannot tell you which way the body is **moving**: a body whose resultant force points down a slope may be sliding down and speeding up, or moving up and slowing down.

One newton is the net force that gives 1 kg an acceleration of 1 m s^{-2} , so $1 \text{ N} = 1 \text{ kg m s}^{-2}$. The first law is just the special case $\Sigma F = 0 \rightarrow a = 0$. For a falling body with no air resistance the only force is mg , so $mg = ma$ and $a = g$ for every mass: a hammer and a feather fall together on the Moon.

Five steps for every $F = ma$ problem

1. Sketch the situation and choose a positive direction (the direction of the acceleration is easiest).
2. Draw a free-body diagram for each body.
3. Write the net force from the diagram, with signs.

4. Set the net force equal to ma .
5. Solve, then check that the answer is sensible.

Worked example 6

A 1200 kg car accelerates from 0 to 15 m s^{-1} in 8.0 s on a level road. The total resistive force is 600 N. Find the driving force.

1. $a = (15 - 0)/8.0 = 1.875 \text{ m s}^{-2}$.
2. Forward positive: $\Sigma F = F - 600$.
3. $F - 600 = 1200 \times 1.875 = 2250$.
4. $F = 2850 \text{ N} \approx 2.9 \text{ kN}$

Lifts and apparent weight

Standing in a lift, two forces act on you: your weight mg down and the normal force F_N from the floor up. A bathroom scale measures F_N (the force you push on it), not your weight. This is your **apparent weight**.

$$\text{up positive: } F_N - mg = ma \rightarrow F_N = m(g + a)$$

$a > 0$ (acceleration up): you feel heavier · $a < 0$ (acceleration down): you feel lighter · free fall $a = -g$: $F_N = 0$, "weightless"

Worked example 7

A 60 kg student stands on a scale in a lift. Find the scale reading when the lift (a) moves up and speeds up at 1.5 m s^{-2} , (b) moves up at a constant 3.0 m s^{-1} , (c) moves up and slows down at 2.0 m s^{-2} .

1. (a) $a = +1.5$: $F_N = 60(9.8 + 1.5) = 678 \text{ N}$
2. (b) $a = 0$: $F_N = 60 \times 9.8 = 588 \text{ N}$ (the speed does not matter, only the acceleration)

3. (c) Moving up but slowing $\rightarrow$ the acceleration is **down**, $a = -2.0$:

$$F_N = 60(9.8 - 2.0) = \mathbf{468\text{ N}}$$
 A scale marked in kg would show

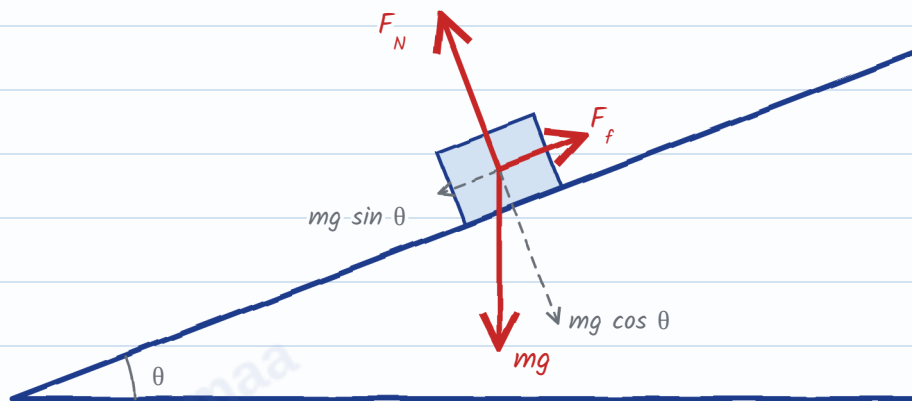
$$468/9.8 = 47.8\text{ kg}.$$

✗ Mistake

Deciding the sign of a from the direction of motion. Only the **acceleration** matters. "Moving down and slowing down" has an upward acceleration, so the student feels **heavier**, just like "moving up and speeding up".

The inclined plane

On a slope at angle θ , choose axes **along** the slope (where the motion is) and **perpendicular** to it. Then split only the weight into components:



The angle between mg and the perpendicular to the slope is θ , the same as the slope angle.

perpendicular: $F_N = mg \cos \theta$ · along the slope (down): $mg \sin \theta$

$$\theta - F_f = ma$$

Frictionless slope: $a = g \sin \theta$ (the same for every mass)

A block on a rough slope stays at rest if $mg \sin \theta \leq \mu_s mg \cos \theta$. Tilt the slope slowly: the block starts to slide when $\tan \theta = \mu_s$. Measuring this angle is a simple way to find μ_s .

Worked example 8

A 5.0 kg block slides down a slope at 30° . $\mu_d = 0.20$, $\mu_s = 0.45$. (a) Find its acceleration. (b) At what angle would the block, placed at rest, just start to slide?

1. $F_N = mg \cos 30^\circ$; $F_f = \mu_d mg \cos 30^\circ$.
2. Down the slope: $mg \sin 30^\circ - \mu_d mg \cos 30^\circ = ma \rightarrow a = g(\sin 30^\circ - 0.20 \cos 30^\circ)$
3. $a = 9.8(0.500 - 0.173) = 3.2 \text{ m s}^{-2}$ (the mass cancels)
4. (b) $\tan \theta = \mu_s = 0.45 \rightarrow \theta = 24^\circ$

Connected bodies

When two or more bodies move together, you have two ways to look at them, and you usually need both:

Whole system: treat all the bodies as one. Forces between them (string tension, contact forces) are **internal** and cancel. Use it to find the **acceleration**.

One body: take a single body on its own. Now the internal force is an **outside** force on it. Use it to find the **tension** or contact force.



free-body diagrams (horizontal forces only):



The 24 N force acts only on the block it is attached to. The 3.0 kg block is pulled by the string alone.

Worked example 9

Find the acceleration of the blocks above and the tension in the string. The table is smooth.

1. Whole system (8.0 kg), net force 24 N: $a = 24/8.0 = 3.0 \text{ m s}^{-2}$
2. 3.0 kg block alone, only T acts horizontally: $T = 3.0 \times 3.0 = 9.0 \text{ N}$
3. Check with the 5.0 kg block: $24 - T = 5.0 \times 3.0 \rightarrow T = 9.0 \text{ N} \checkmark$

✗ Mistake

Putting the 24 N force on the 3.0 kg block too. A force acts only on the body it touches. The rear block is pulled by the string, and the string tension (9.0 N) is less than 24 N, because part of the 24 N is used to accelerate the front block.

Lifting one body below another

The same two steps work vertically. A force F lifts a body of mass m_1 , and a string below it holds a second body of mass m_2 . Take up as positive.

$$\text{whole system: } F - (m_1 + m_2)g = (m_1 + m_2)a$$

$$\text{lower body alone: } T - m_2g = m_2a \rightarrow T = m_2F/(m_1 + m_2)$$

Worked example 9b

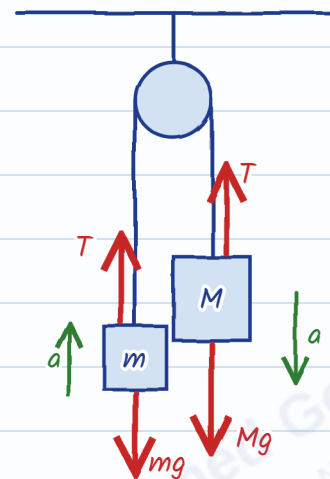
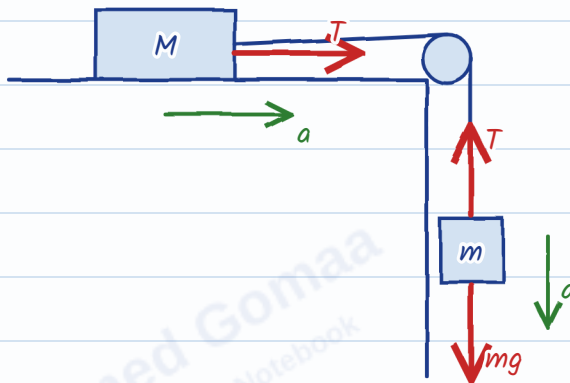
A crane lifts a 4.0 kg box with a force of 100 N. A 2.0 kg box hangs from it on a light string. Find the acceleration and the tension in the string.

1. Whole system (6.0 kg): $a = (100 - 6.0 \times 9.8)/6.0 = 41.2/6.0 = 6.9 \text{ m s}^{-2}$ upwards.
2. Lower box alone: $T - 2.0 \times 9.8 = 2.0 \times 6.87 \rightarrow T = 33 \text{ N}$.
3. Check: $T = m_2 F / (m_1 + m_2) = 2.0 \times 100 / 6.0 = 33 \text{ N} \checkmark$. The string carries only the share of F needed for the lower box.

Pulleys: the two set-ups

A light, smooth (frictionless) pulley only changes the **direction** of the string. The tension is the same on both sides, and the two bodies have accelerations of the same size. There are two set-ups you must know.

Set-up 1: table + hanging mass



Set-up 2: Atwood machine

Same tension on both sides of a light, smooth pulley. Each body has acceleration a , in the direction it moves.

Set-up 1: block on a table pulled by a hanging mass

Take the positive direction for each body as the way it moves: right for the block on the table, down for the hanging mass.

hanging mass: $mg - T = ma$ · block on table: $T - F_f = Ma$
 add them: $a = (mg - F_f) / (M + m)$ smooth table: $F_f = 0$

Worked example 10

A 4.0 kg block on a table is joined over a pulley to a hanging 1.0 kg mass. (a) The table is smooth: find a and T . (b) Now $\mu_s = 0.30$ and $\mu_d = 0.20$. Does the block move? (c) The hanging mass is changed to 2.0 kg. Find a and T .

1. (a) $a = mg / (M + m) = 1.0 \times 9.8 / 5.0 = 1.96 \text{ m s}^{-2}$; $T = Ma = 4.0 \times 1.96 = 7.8 \text{ N}$
2. (b) The pull is $mg = 9.8 \text{ N}$. The largest static friction is $\mu_s Mg = 0.30 \times 39.2 = 11.8 \text{ N}$. Since $9.8 \text{ N} < 11.8 \text{ N}$, the block stays at rest (friction = 9.8 N , $T = 9.8 \text{ N}$).
3. (c) Pull = $2.0 \times 9.8 = 19.6 \text{ N} > 11.8 \text{ N}$, so it slides. $F_f = \mu_d Mg = 0.20 \times 39.2 = 7.84 \text{ N}$.
4. $a = (19.6 - 7.84) / 6.0 = 1.96 \text{ m s}^{-2}$; $T = m(g - a) = 2.0(9.8 - 1.96) = 15.7 \text{ N}$

Set-up 2: the Atwood machine

Two masses hang on either side of a pulley. The heavier mass M goes down, the lighter mass m goes up.

M (down +): $Mg - T = Ma$ · m (up +): $T - mg = ma$
 add: $a = (M - m)g / (M + m)$ · then $T = 2Mmg / (M + m)$

Worked example 11

Masses of 2.0 kg and 3.0 kg hang over a smooth pulley and are released from rest. Find (a) the acceleration, (b) the tension, (c) the time for the 3.0 kg mass to fall 1.0 m.

1. (a) $a = (3.0 - 2.0) \times 9.8 / 5.0 = 1.96 \text{ m s}^{-2}$

2. (b) $T = m(g + a) = 2.0(9.8 + 1.96) = 23.5 \text{ N}$

3. (c) $s = \frac{1}{2}at^2 \rightarrow t = \sqrt{2 \times 1.0 / 1.96} = 1.0 \text{ s}$

Why is the tension between mg and Mg ?

The light mass accelerates **up**, so T must be **more** than its weight (19.6 N). The heavy mass accelerates **down**, so T must be **less** than its weight (29.4 N). 23.5 N fits. If the masses were equal, $a = 0$ and $T = mg$. The Atwood machine "slows gravity down": the small acceleration is easy to time, so it is a simple way to measure g in a school lab.

✗ Mistake

"The tension equals the weight of the hanging mass." Only if nothing accelerates. When the hanging mass accelerates down, $T < mg$. Always write $F = ma$ for the hanging mass on its own.

Pulley problems in four lines

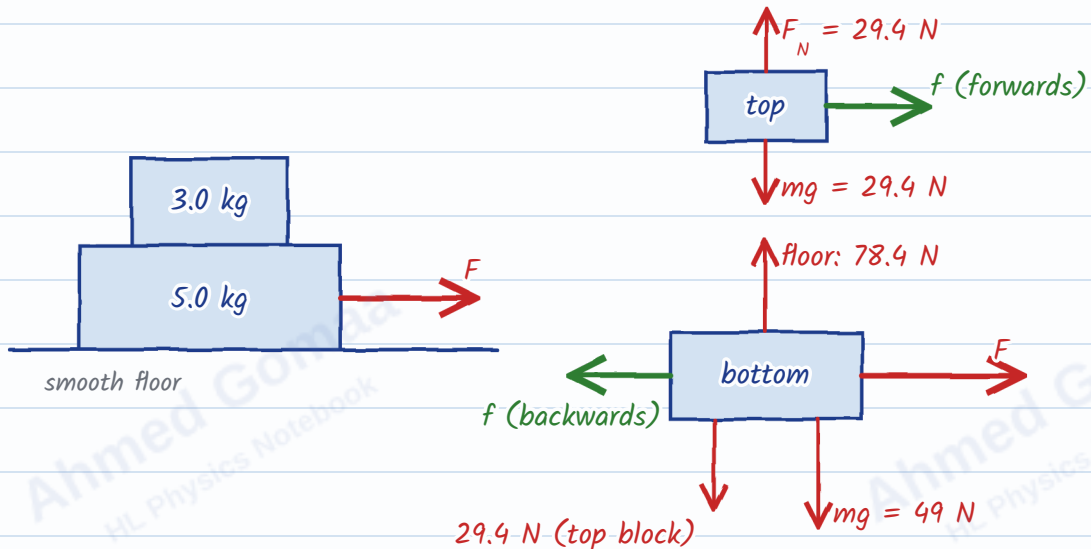
1. Mark the direction each body moves; that is its positive direction.
2. Free-body diagram for each body (same T both sides).
3. Write $\Sigma F = ma$ for each body.
4. Add the equations: T cancels and gives a . Put a back in to get T .

One block on another

This set-up comes up in exams again and again, so learn it well. A block rests on top of another block on a **smooth** floor, and a horizontal force F

pulls the **bottom** block. The only thing that can drag the top block along is **friction** between the two blocks.

Which way does that friction point? As the bottom block moves off, the top block tends to stay where it is (inertia), so it tends to slide **backwards** relative to the bottom block. Friction opposes this, so friction on the top block points **forwards**. By the third law, the top block pushes friction on the bottom block **backwards**, with the same size.



The two green arrows are a third-law pair: equal friction forces, opposite directions, on different blocks. The floor is smooth, so there is no friction under the bottom block.

Worked example 12

A 3.0 kg block sits on top of a 5.0 kg block on a smooth horizontal floor. Between the blocks $\mu_s = 0.50$ and $\mu_d = 0.40$. A horizontal force F is applied to the **bottom** block.

(a) Find the largest F for which the two blocks move together without sliding.

(b) F is now 60 N. Find the acceleration of each block.

1. (a) The normal force between the blocks is the weight of the top block: $3.0 \times 9.8 = 29.4$ N. So the largest friction on the top block is $\mu_s \times 29.4 = 0.50 \times 29.4 = 14.7$ N.

2. Top block: its only horizontal force is friction, so its largest acceleration is $a = 14.7/3.0 = 4.9 \text{ m s}^{-2}$.
3. Moving together, both blocks have this acceleration. Treat them as one body of 8.0 kg: $F = 8.0 \times 4.9 = 39 \text{ N}$ (39.2 N).
4. Check with the bottom block alone: $F - 14.7 = 5.0 \times 4.9 \rightarrow F = 39.2 \text{ N} \checkmark$
5. (b) $60 \text{ N} > 39.2 \text{ N}$, so the blocks slide over each other. Now the friction is dynamic: $f = \mu_d \times 29.4 = 0.40 \times 29.4 = 11.8 \text{ N}$.
6. Top block: $a = 11.8/3.0 = 3.9 \text{ m s}^{-2}$ forwards.
7. Bottom block: $60 - 11.8 = 5.0a \rightarrow a = 9.6 \text{ m s}^{-2}$ forwards.
8. Both move forwards, but the bottom block gets ahead: seen from the bottom block, the top block slides backwards.

✗ Mistake

Treating the two blocks as one body in part (b). You can only do that while they move **together**. Once they slide, they have different accelerations, so you must write $F = ma$ for each block separately, with the dynamic friction value.

Method for stacked blocks

1. Find the largest friction: $\mu_s \times (\text{weight of the top block})$.
2. Find the largest acceleration of the block that is pulled **only** by friction.
3. Together: $F_{\max} = (\text{total mass}) \times \text{that acceleration}$.
4. If F is larger: they slide; use μ_d and $F = ma$ for each block on its own.

Writing the explanations

Many marks on forces are for free-body diagrams and short explanations. Markschemes look for the same key words every time: name each force, say which body exerts it, and say what the **resultant force** does.

“Draw a free-body diagram for the body” (2–3 marks)

- ✓ One arrow for each real force, and every arrow **touches the body**. Friction is drawn on the surface where the two bodies touch.

- ✓ Each arrow has an accepted label: weight (W , mg , F_g), normal force (F_N , N , R), friction (F_f), tension (T), drag (F_d), buoyancy (F_b).

- ✓ Directions and lengths fit the motion: balanced forces get equal arrows (constant velocity, terminal speed); a larger force gets a longer arrow (tension longer than weight at the bottom of a swing).

One extra arrow such as a “driving force”, “force of motion” or “centripetal force” loses the mark.

“State Newton’s third law” (1 mark) / “Explain how the third law applies when A hits B” (2 marks)

- ✓ If body A exerts a force on body B, then **B exerts a force on A**.

- ✓ The two forces are **equal in size**.

- ✓ They are **opposite in direction**.

Name both bodies every time: “the racket exerts a force on the ball, and the ball exerts an equal and opposite force on the racket”.

“Outline, using Newton’s third law, how the lift (thrust) on a vehicle is produced” (2–3 marks)

- ✓ The blades (fan, engine) exert a force on the air (gas) in one direction, for example **downwards** or backwards.
- ✓ By Newton's third law the air exerts an **equal and opposite force** on the blades, so on the vehicle.
- ✓ By Newton's second law this force changes the velocity of the vehicle (it speeds up, slows down, or holds it up). When it hovers, the resultant force is **zero**.

Give the direction of the force on the air. The momentum version is in the momentum notes.

"Explain, with reference to the forces and where they come from, why the vertical forces are balanced" (2-3 marks)

- ✓ Weight: the gravitational force **from the Earth**, downwards.
- ✓ Normal force: the push **from the ground** (snow, floor), upwards.
- ✓ The body has **no vertical acceleration** (it stays on the ground), so by Newton's first law the resultant vertical force is zero.

For "Describe the motion with reference to Newton's first (second) law": name the force (or say there is none) and say what happens: "no resultant force, so constant velocity" or "the component of weight acts down the slope, so it slows down".

"Explain why the body speeds up once it starts to slide" (3 marks)

- ✓ The coefficient of static friction is larger than the coefficient of dynamic friction: $\mu_s > \mu_d$.
- ✓ So once it slides, the dynamic friction force is **smaller** than the maximum static friction, which the push only just matched.
- ✓ Now there is a **resultant (net) force** forwards, so by $F = ma$ the body accelerates.

For "Will the block move?": work out the maximum static friction $\mu_s F_N$, compare it with the push (or $mg \sin \theta$) and write the conclusion.

"Explain, by reference to the forces, how the body reaches terminal speed" (2–3 marks)

- ✓ A drag force acts against the velocity, and it increases with speed.
- ✓ So the resultant force gets smaller and the acceleration falls.
- ✓ Terminal speed is reached when the **resultant force is zero**: falling, drag + buoyancy = weight; rising, buoyancy = weight + drag.

"State the initial acceleration" of a falling drop: write g or 9.8 m s^{-2} , not just "gravity". For a body released in a liquid, include the buoyancy.

"Explain the origin of the buoyancy force" (2 marks)

- ✓ The pressure in a fluid increases with depth.
 - ✓ So the fluid pushes up on the bottom of the body harder than it pushes down on the top: there is a net upward force.
- Another answer that scores: the buoyancy equals the **weight of the fluid displaced**, which is larger than the weight of the body (so it rises).

Common mistakes

✗ Ten traps in A.2 forces

1. Labelling the weight "gravity" or "g". Write weight, W , mg or F_g .
2. Adding a "driving force" or "force of motion" to a body that is moving.
Only real forces from other bodies go on the diagram.
3. Drawing arrows that do not touch the body, or friction away from the contact surface.
4. Third law: "the same force" or "the reaction" with no bodies named.
Say A on B and B on A, equal in size, opposite in direction.
5. Calling the weight and the normal force a third-law pair. They act on the same body.
6. Writing a bald "by Newton's second law" or " $F = ma$ ". Name the force and say what it does to the motion.
7. Saying a hovering or steady body has a resultant force upwards.
Constant velocity means the resultant force is zero.
8. Using $\mu_s F_N$ for a sliding body. Once it slides, friction is $\mu_d F_N$, which is smaller.
9. Forgetting buoyancy in a liquid: at terminal speed drag + buoyancy = weight. On a rising body the drag points down.
10. Saying a falling body speeds up "until the forces cancel" with no reason. Say the drag increases with speed until the resultant force is zero.

Exam habits

Free-body diagram first, one body at a time. Choose the positive direction as the direction of acceleration. Write $\Sigma F = ma$ with every force and its sign. Whole system for a ; one body for T .
Give answers to 2 or 3 significant figures with units.

Practise next

You have read the notes. Now practise, in this order:

1. **Levelled practice**: first a ★ at Level 2 in every topic (the goal), then Levels 3 and 4 for a 6 or 7. Hints and worked answers help you as you go.
2. **IB-format practice** ([Paper 1A](#) · [Paper 2](#)): timed, one attempt, and your report goes to your teacher. Do these when the practice goal is reached.

© 2026 Ahmed Gomaa · HL Physics Notebook · Original material. All rights reserved. Free for personal study. Teachers: please [ask](#) before sharing or printing for a class.

Not affiliated with or endorsed by the International Baccalaureate Organization.

© 2026 Ahmed Gomaa · All rights reserved · Free for personal study · Not affiliated with the IBO