

D.1 Gravitational fields

Ahmed Gomaa · IB DP Physics SL + HL (sections marked HL are HL only)
· uses A.2 (circular motion) and A.3 (energy) · original worked examples

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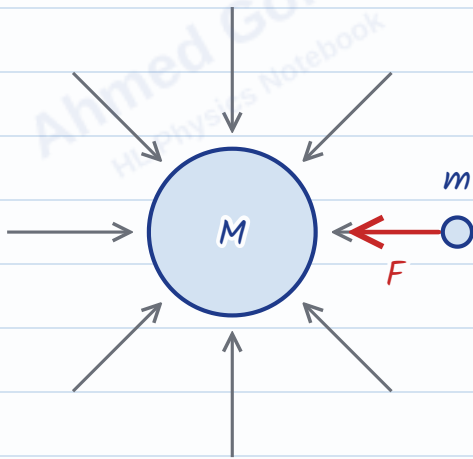
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Pen colours: blue = bodies and diagrams, red = forces, laws and key steps, green = answers and motion, pencil = construction lines. Boxes: red-edged box with "learn it word for word" = a definition the markscheme wants exactly; **Why?** = the reasoning; **Mistake** = what students get wrong; yellow note = exam tip or important note. Use $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$ unless told otherwise; for the Earth, $M = 6.0 \times 10^{24} \text{ kg}$ and $R = 6.4 \times 10^6 \text{ m}$.

The big picture: a field of force

In A.2 the weight of a body was mg , with $g = 9.8 \text{ N kg}^{-1}$. D.1 explains where that force comes from: **every mass attracts every other mass.** A

mass creates a **gravitational field** in the space around it, and any other mass placed in that field feels a force. The same idea, with charge instead of mass, gives the electric field of D.2.



A mass M makes a field in the space around it. A small mass m placed in the field feels a force.

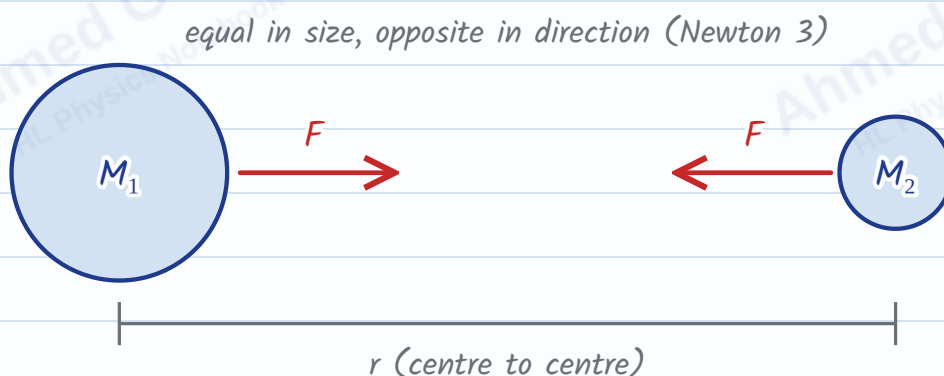
field strength = force per unit mass
 $g = F/m \text{ (N kg}^{-1} = \text{m s}^{-2}\text{)}$

A mass makes a field; a small test mass placed in it feels a force towards the mass.

Newton's law of gravitation

Newton's universal law of gravitation $F = G m_1 m_2 / r^2$

for bodies treated as **point masses**; r is the distance between their centres; $G = 6.67 \times 10^{-11} \text{ N m}^2 \text{ kg}^{-2}$. The force is always **attractive** and acts along the line joining the two masses.



Each body pulls the other with a force of the same size (Newton's third law), even when the masses are very different.

When can a real body be treated as a point mass? A sphere of uniform density acts as if all its mass were at its centre, so r is measured centre to centre. Any body can also be treated as a point mass if it is very small compared with the distance between the bodies (a satellite seen from a planet).

Exam tip: "State the conditions..." (2 marks)

The planet is **spherical** (and of uniform density), so it acts as a point mass at its centre; the satellite is **much smaller** than the planet / the distance between them.

Worked example 1

A star of mass 3.0×10^{30} kg and a planet of mass 4.0×10^{24} kg are 2.0×10^{11} m apart. Find the gravitational force between them and the acceleration of the planet.

1. $F = Gm_1m_2/r^2 = 6.67 \times 10^{-11} \times 3.0 \times 10^{30} \times 4.0 \times 10^{24} / (2.0 \times 10^{11})^2$
 $= 2.0 \times 10^{22}$ N.

2. The planet's acceleration $a = F/m = 2.0 \times 10^{22} / 4.0 \times 10^{24} = 5.0 \times 10^{-3}$ m s⁻² (towards the star). The star feels the same force but, with its huge mass, hardly accelerates.

Worked example 2

The distance between two bodies is tripled. What happens to the force between them?

1. $F \propto 1/r^2$, so tripling r divides F by $3^2 = 9$.

2. The force becomes **one ninth** of its value. (Doubling r would give one quarter.)

Gravitational field strength

Gravitational field strength g at a point · learn it word for word the force per unit mass experienced by a small point mass at that point.

$g = F/m = GM/r^2$ unit $N\ kg^{-1}$ ($= m\ s^{-2}$); g is a **vector** pointing towards the mass M .

Why is g the same as the acceleration of free fall?

The only force on a falling body is its weight $F = mg$. Newton's second law gives $a = F/m = g$. So the field strength in $N\ kg^{-1}$ and the free-fall acceleration in $m\ s^{-2}$ are the same number.

✗ Mistake

Writing "gravity" in a definition. Write **force per unit mass on a small point mass**: "the strength of gravity" earns nothing.

Worked example 3

A planet has 8 times the mass of the Earth and 4 times its radius.

Find the gravitational field strength at its surface (g on Earth = $9.8\ N\ kg^{-1}$).

1. Write the formula in Earth terms: $g = G(8M)/(4R)^2 = (8/16) \times GM/R^2$.

2. $g = 0.5 \times 9.8 = 4.9\ N\ kg^{-1}$. Do not forget to **square** the 4.

Worked example 4

Find the gravitational field strength 400 km above the Earth's surface.

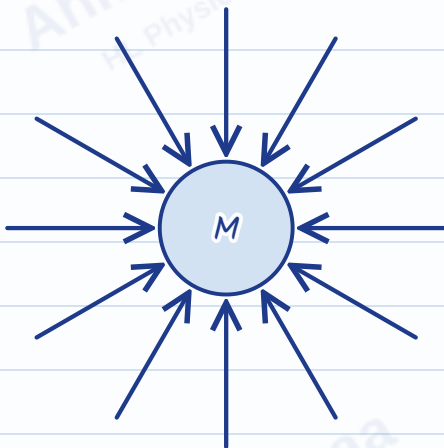
1. Add the height to the radius: $r = 6.4 \times 10^6 + 0.4 \times 10^6 = 6.8 \times 10^6$ m.

2. $g = GM/r^2 = 6.67 \times 10^{-11} \times 6.0 \times 10^{24} / (6.8 \times 10^6)^2 = 8.7 \text{ N kg}^{-1}$.

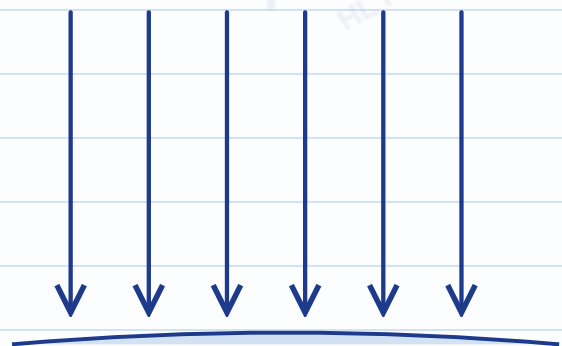
Gravity is far from zero up there: about 90% of its surface value.

Field lines

A field line shows the direction of the force on a small mass. For gravity the lines always point **towards** the mass. Where the lines are closer together, the field is stronger.



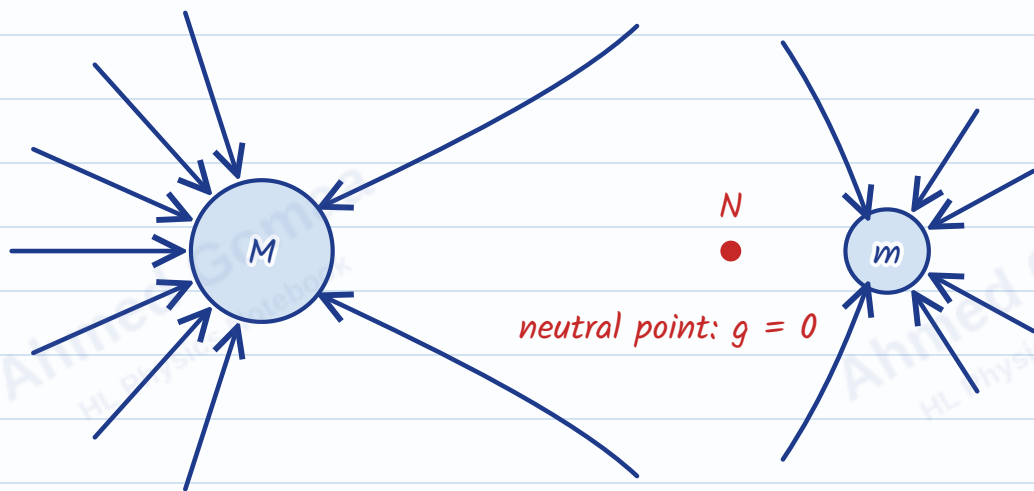
radial: lines spread out, g weaker



surface of a planet

close to the surface: uniform field

Around a sphere the field is radial and weakens with distance; close to a planet's surface it is (nearly) uniform: parallel, equally spaced lines.



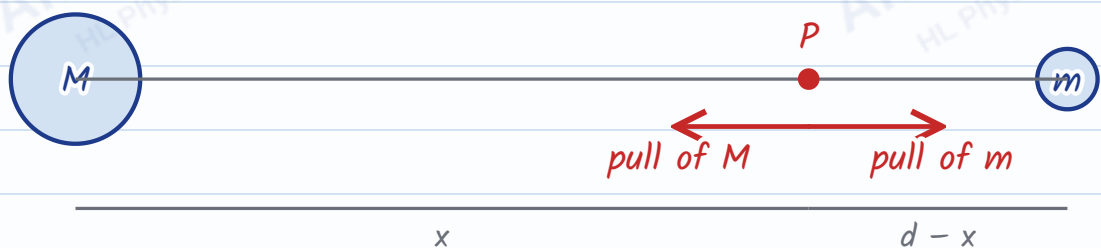
Two masses: the lines from both bend towards the masses. At the neutral point N between them the two pulls cancel.

Exam tip: sketching field lines

Draw arrows **towards** the mass, **perpendicular to the surface**, never crossing, with spacing that grows with distance for a radial field and stays equal for a uniform field.

Where the net field is zero

Between two masses, each pulls a test mass towards itself. Somewhere on the line joining them the two pulls are equal in size and opposite in direction: the **net field is zero**. The pull from the left equals the pull from the right. This point is always **closer to the smaller mass**.



$$g = 0 \text{ where } GM/x^2 = Gm/(d - x)^2$$

At P the two pulls balance.

Net field zero at distance x from M : $GM/x^2 = Gm/(d-x)^2$ so
 $x/(d-x) = \sqrt{M/m}$

Worked example 5

A planet of mass 6.0×10^{24} kg has a moon of mass $1/64$ of its mass. Their centres are 4.5×10^8 m apart. (a) Find where the net gravitational field is zero. (b) Find the gravitational potential there (HL). (c) Find the potential energy of a 3000 kg probe placed there (HL).

1. (a) $GM/x^2 = G(M/64)/(d-x)^2$, so $x/(d-x) = \sqrt{64} = 8$.
2. $x = 8(d-x) \rightarrow 9x = 8d \rightarrow x = 8 \times 4.5 \times 10^8 / 9 = 4.0 \times 10^8$ m from the planet (0.5×10^8 m from the moon).
3. (b) Potential is a scalar: add the two values. $V = -GM/x - G(M/64)/(d-x) = -1.0 \times 10^6 - 0.13 \times 10^6 = -1.1 \times 10^6$ J kg⁻¹.
4. (c) $E_p = mV = 3000 \times (-1.1 \times 10^6) = -3.4 \times 10^9$ J.

Important note: zero field on a $V-r$ graph

The field is minus the gradient of the potential-distance graph (see [Field and potential](#)). So the net field is zero exactly where the $V-r$ graph is flat: its tangent is horizontal. Between a planet and its moon this is the **top of the hump** of the $V-r$ curve. Questions ask it both ways: as a word problem (equal pulls) and from a graph (zero gradient).

✗ Mistake

Adding the two field strengths at the zero point. They point in **opposite** directions, so you set them **equal**; you add them only when finding the potential (a scalar).

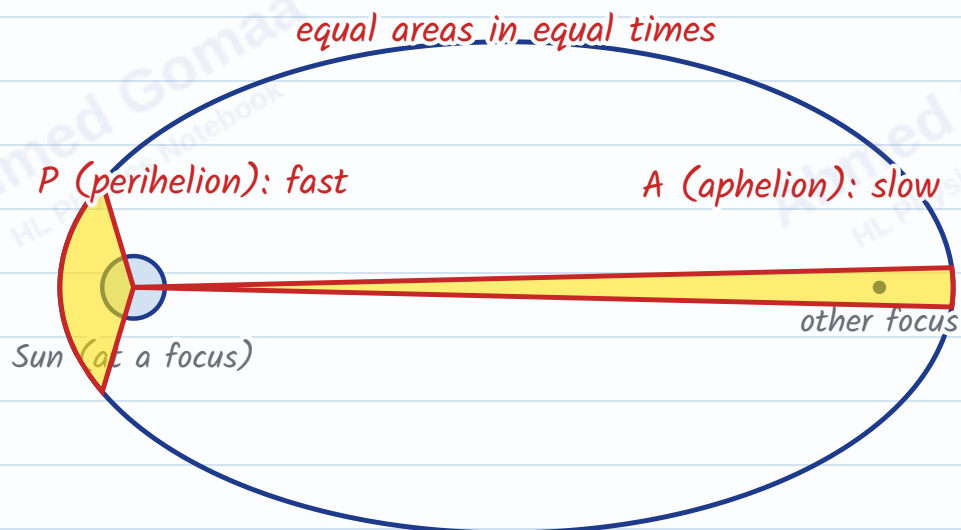
Kepler's three laws

Kepler found three rules from careful observations of the planets, decades before Newton explained them with his law of gravitation.

Kepler's first law: each planet moves on an **ellipse** with the Sun at one **focus**.

Kepler's second law: the line joining a planet to the Sun sweeps out **equal areas in equal times**.

Kepler's third law: the square of the orbital period is proportional to the cube of the semi-major axis: $T^2 \propto a^3$ (for a circular orbit $a = r$).



Equal areas in equal times: near the Sun (perihelion) the planet must move faster to sweep the same area.

✗ Mistake

Saying "equal arc lengths" (it is equal areas), putting the Sun at the **centre** of the ellipse (it is at a **focus**), or writing $T \propto r^3$ (it is $T^2 \propto r^3$).

Worked example 6

Two moons orbit the same planet in circular orbits. Moon A has radius $4.0 \times 10^8 \text{ m}$ and period 2.0 days. Moon B has radius $1.0 \times 10^9 \text{ m}$. Find the period of moon B.

1. Same central mass, so T^2/r^3 is the same for both: $T_B = T_A \times (r_B/r_A)^{(3/2)}$.
2. $r_B/r_A = 2.5$, and $2.5^{1.5} = 3.95$.
3. $T_B = 2.0 \times 3.95 = 7.9 \text{ days}$.

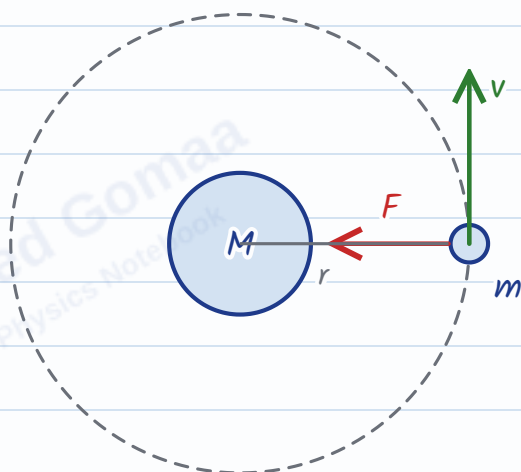
Worked example 7

A comet is $1.0 \times 10^{11} \text{ m}$ from the Sun at its closest point and $4.0 \times 10^{11} \text{ m}$ at its furthest. Find the ratio (speed at the closest point)/(speed at the furthest point).

1. In a short time t the areas swept are about $\frac{1}{2} \times (vt) \times \text{distance}$.
Equal areas: $\frac{1}{2} v_P t \times r_P = \frac{1}{2} v_A t \times r_A$.
2. $v_P/v_A = r_A/r_P = 4.0 \times 10^{11}/1.0 \times 10^{11} = 4$. The comet is 4 times faster near the Sun.

Orbits: $v = \sqrt{GM/r}$ and $T^2 = 4\pi^2 r^3/GM$

For a circular orbit the only force on the satellite is gravity, and it points to the centre. So **gravity provides the centripetal force**. This one idea gives every orbit formula, and deriving Kepler's third law from it is one of the most common IB questions.



gravity provides the centripetal force:

$$GMm/r^2 = mv^2/r$$

$$v = \sqrt{GM/r}$$

$$T^2 = (4\pi^2/GM) r^3$$

Gravity is the centripetal force.

Show that (learn this derivation)

Show that the period T of a satellite in a circular orbit of radius r around a body of mass M is given by $T^2 = 4\pi^2 r^3 / (GM)$.

1. Gravity provides the centripetal force: $GMm/r^2 = mv^2/r$.
2. Cancel m and one r : $v^2 = GM/r$, so $v = \sqrt{GM/r}$.
3. One orbit is a distance $2\pi r$ in a time T : $v = 2\pi r/T$, so $v^2 = 4\pi^2 r^2 / T^2$.
4. Equate: $4\pi^2 r^2 / T^2 = GM/r \rightarrow T^2 = 4\pi^2 r^3 / (GM)$. This is Kepler's third law: $T^2 \propto r^3$ with the constant $4\pi^2 / GM$.

How the marks go (2-3 marks)

✓ Clear evidence that the **centripetal force is equated to the gravitational force** ($GMm/r^2 = mv^2/r$ or $m\omega^2 r$).

✓ $v = 2\pi r/T$ (or $\omega = 2\pi/T$) substituted.

✓ Rearranged to the result.

Do not start from an equation of simple harmonic motion or from the answer.

Worked example 8

A geostationary satellite orbits above the equator with a period of 24 hours. Find its orbit radius, its height above the surface and its speed.

1. $T = 24 \times 3600 = 86\,400 \text{ s}$. From $T^2 = 4\pi^2 r^3 / GM$: $r^3 = GMT^2 / 4\pi^2$.
2. $r = (6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 86\,400^2 / 4\pi^2)^{1/3} = 4.2 \times 10^7 \text{ m}$.
3. Height $= r - R = 4.2 \times 10^7 - 0.64 \times 10^7 = 3.6 \times 10^7 \text{ m}$ (36 000 km).
4. $v = 2\pi r / T = 3.1 \times 10^3 \text{ m s}^{-1}$. It stays above the same point on the equator, so a fixed dish can point at it.

Worked example 9

A moon orbits a planet with radius $6.0 \times 10^8 \text{ m}$ and period 3.0 days. Find the mass of the planet.

1. Rearrange Kepler's third law: $M = 4\pi^2 r^3 / (GT^2)$, with $T = 3.0 \times 86\,400 \text{ s}$.
2. $M = 4\pi^2 \times (6.0 \times 10^8)^3 / (6.67 \times 10^{-11} \times (2.59 \times 10^5)^2) = 1.9 \times 10^{27} \text{ kg}$. Watching a moon is how we "weigh" a planet.

Worked example 10

Find the orbital speed of a satellite 500 km above the Earth, and of one that just grazes the surface. Find the period of the grazing orbit.

1. $r = 6.9 \times 10^6 \text{ m}$: $v = \sqrt{GM/r} = 7.6 \times 10^3 \text{ m s}^{-1}$.
2. Grazing, $r = 6.4 \times 10^6 \text{ m}$: $v = 7.9 \times 10^3 \text{ m s}^{-1}$; lower orbits are faster.
3. $T = 2\pi r / v = 2\pi \times 6.4 \times 10^6 / 7.9 \times 10^3 = 5.1 \times 10^3 \text{ s} \approx 85 \text{ minutes}$.

× Mistake

Using the height above the surface as r . In every formula r is measured from the **centre** of the planet: $r = R + h$. Markschemes give [1] max when the height is ignored.

Gravitational potential energy (HL)

Gravitational potential energy E_p of a system · learn it word for word the work done to assemble the system from infinite separation of the components of the system.

$E_p = -G m_1 m_2 / r$ where r is the separation between the centres of mass of the two bodies. Unit J. E_p belongs to the pair of bodies, not to one of them.

Worked example 11

Find the change in gravitational potential energy when a 1200 kg satellite is taken from the Earth's surface to a height of 600 km.

1. At the surface: $E_p = -GMm/R = -6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times$

$1200 / 6.4 \times 10^6 = -7.50 \times 10^{10} \text{ J.}$

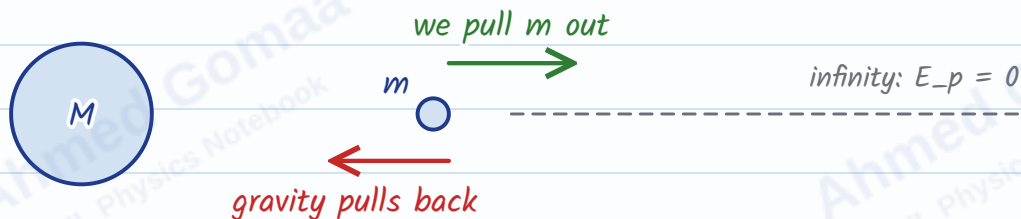
2. At $r = 7.0 \times 10^6 \text{ m}$: $E_p = -6.86 \times 10^{10} \text{ J.}$

3. $\Delta E_p = (-6.86 \times 10^{10}) - (-7.50 \times 10^{10}) = +6.4 \times 10^9 \text{ J.}$ Positive: the energy goes up as the satellite moves away, even though both values are negative.

✗ Mistake

Using mgh for a height of hundreds of kilometres. g changes with height, so mgh is wrong for large heights (markschemes award [0]). Use $-GMm/r$ at both points and subtract.

Why gravitational potential is always negative (HL)



Gravity always ATTRACTS, so we must do positive work to take m out to infinity, where $E_p = 0$.

So everywhere closer than infinity, E_p and V are below 0: **NEGATIVE**.

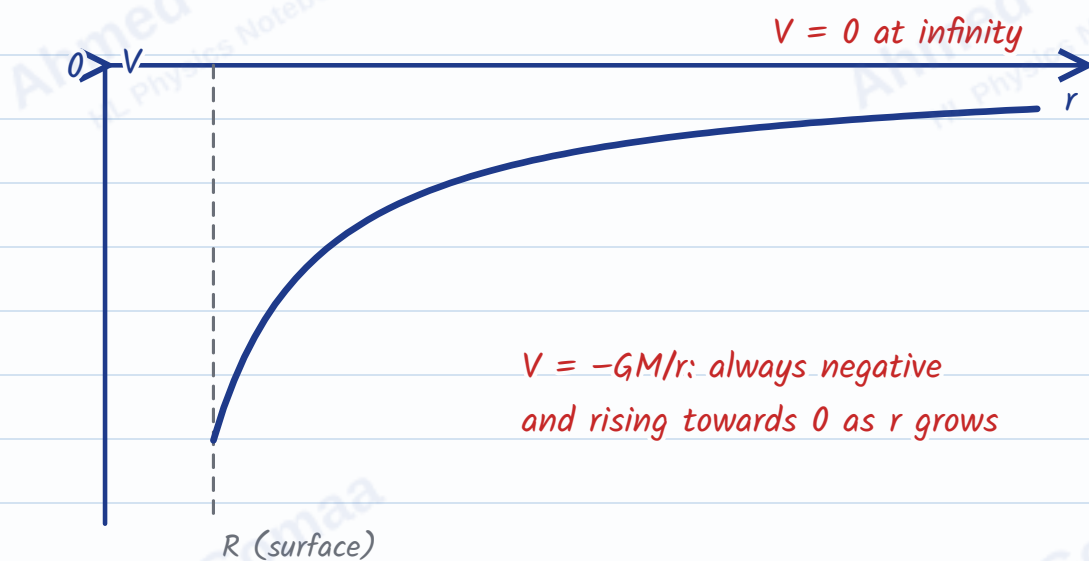
Gravity attracts, so it takes positive work to pull the mass out to infinity, where the energy is zero.

Important note

Gravitational potential and gravitational potential energy are **always negative**, because gravity is **always attractive**.

1. Zero is defined at **infinity**.
2. To take a mass from any point out to infinity we must do **positive work** against the attraction.
3. So the mass starts with **less energy** than at infinity: less than zero, which means **negative**.
4. As r increases, V rises towards zero (it gets **less negative**), but it never becomes positive.

Compare D.2: like charges **repel**, so the electric potential of a positive charge is positive.



The “potential well”: $V = -GM/r$ is below zero everywhere and rises towards zero far away.

How the marks go: “Explain why the gravitational potential is negative” (2 marks)

- ✓ The potential is **defined as zero at infinity**.
- ✓ The force is **attractive**, so **positive work must be done** (energy supplied) to move a mass to infinity: the potential everywhere else is less than zero.

Gravitational potential (HL)

Gravitational potential V_g at a point · learn it word for word

the work done per unit mass in bringing a small mass from infinity to that point.

$V_g = -GM/r$ unit $J\ kg^{-1}$; V_g is a scalar. A mass m at that point has $E_p = mV_g$.

× Mistake

Leaving out “per unit mass”, or saying “from the point to infinity”. Both lose the mark. And never drop the minus sign: “must see negative sign”.

Worked example 12

The gravitational potential at a point near a planet is $-2.5 \times 10^7\ J\ kg^{-1}$. Find the potential energy of a 500 kg body placed there.

1. $E_p = mV = 500 \times (-2.5 \times 10^7) = -1.25 \times 10^9\ J$.

Worked example 13

Point P is $4.0 \times 10^5\ m$ from the centre of an asteroid of mass $5.0 \times 10^{12}\ kg$ and $1.5 \times 10^5\ m$ from the centre of a smaller asteroid of mass $8.0 \times 10^{10}\ kg$. Find the gravitational potential at P.

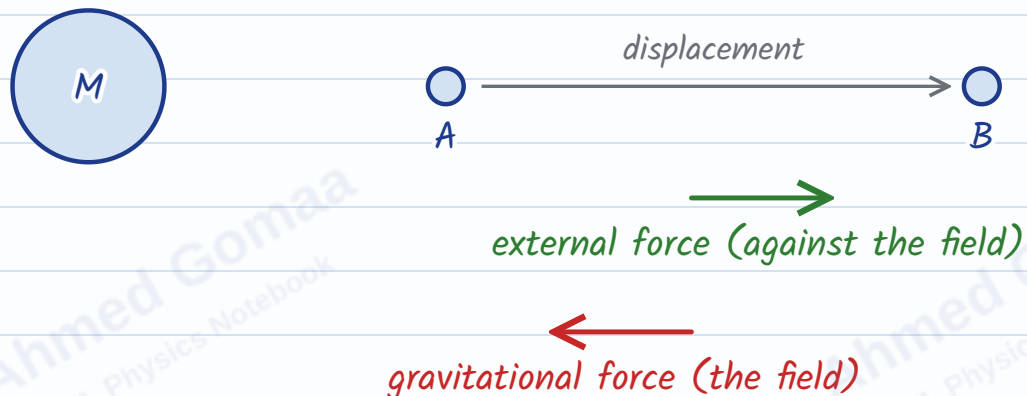
1. Potential is a scalar: find each one, then add (no directions).

2. $V_1 = -GM_1/r_1 = -6.67 \times 10^{-11} \times 5.0 \times 10^{12} / 4.0 \times 10^5 = -8.34 \times 10^{-4}\ J\ kg^{-1}$; $V_2 = -GM_2/r_2 = -3.56 \times 10^{-5}\ J\ kg^{-1}$.

3. $V = V_1 + V_2 = -8.7 \times 10^{-4}\ J\ kg^{-1}$.

Work done in a gravitational field (HL)

$W = m\Delta V_g$ the work done *against the field* (by an external force, at a small constant speed) to move a mass m between two points. It does not depend on the path.



Moving from A to B, away from the planet: the external force and the gravitational force point opposite ways.

	Work done <i>AGAINST</i> the field (by an external force)	Work done <i>BY</i> the field (by gravity)
Equation	$W = m\Delta V = m(V_B - V_A)$	$W = -m\Delta V$
Moving away from the planet (V rises)	positive: we supply energy	negative
Moving towards the planet (V falls)	negative	positive: gravity speeds the body up

Worked example 14

The gravitational potential at A is $-6.0 \times 10^7 \text{ J kg}^{-1}$ and at B it is $-2.0 \times 10^7 \text{ J kg}^{-1}$. Find the work done in moving a 150 kg probe from A to B

(a) by an external agent, (b) by the gravitational force.

1. (a) $W = m(V_B - V_A) = 150 \times (-2.0 \times 10^7 - (-6.0 \times 10^7)) = +6.0 \times 10^9 \text{ J}$ (against the field).

2. (b) The gravitational force does the opposite: $-6.0 \times 10^9 \text{ J}$.

✗ Mistake

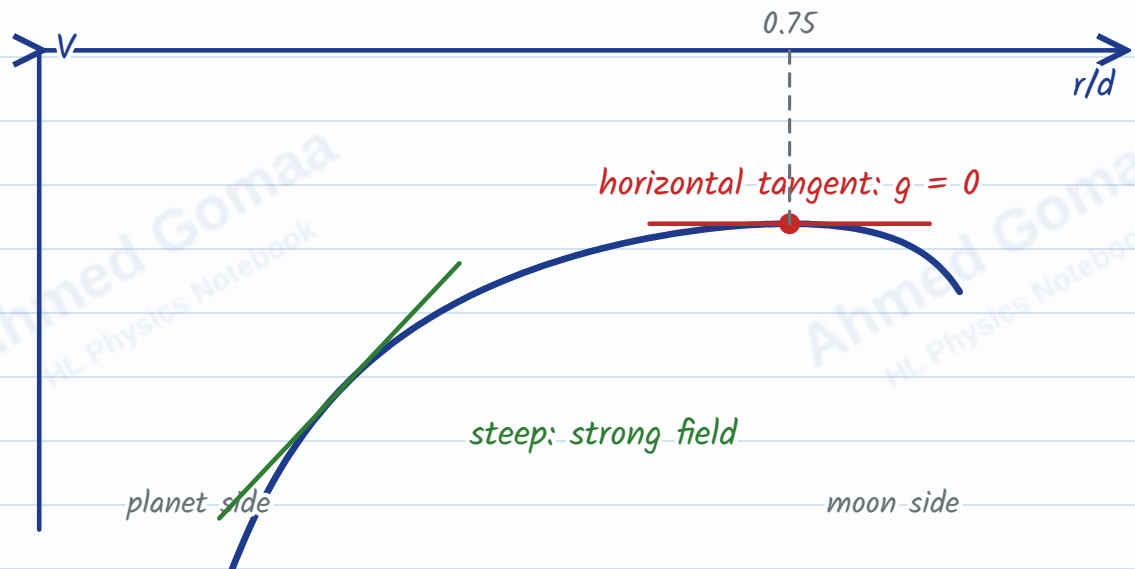
Using $W = F \times \text{distance}$ with the force at A. The force changes from A to B, so $F \times d$ gives the wrong answer; use $W = m\Delta V$.

Field and potential: $g = -\Delta V / \Delta r$ (HL)

Move a mass m a small distance Δr against the field: the work is $mg\Delta r$, and it is also $m\Delta V$. So $g = \Delta V / \Delta r$ in size. With the direction (the field points the way V decreases):

$g = -\Delta V_g / \Delta r$ the field strength is **minus the gradient** (minus the slope of the tangent) of the graph of V against r .

Steep V - r graph $\rightarrow$ strong field. Flat V - r graph (zero gradient) $\rightarrow$ zero field.



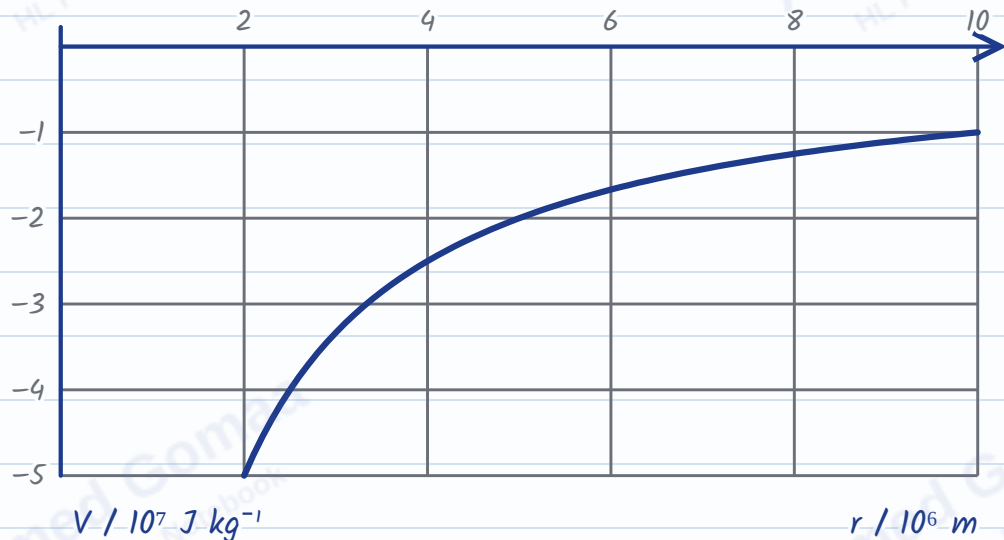
Potential between a planet and its moon. At the top of the hump the tangent is horizontal: the net field is zero there ($r = 0.75 d$ in this sketch).

Important note

The **net field is zero where the $V-r$ graph has zero gradient.** To read where $g = 0$ from a graph, find where the tangent is horizontal (the maximum of V between two bodies). To read g at any point, draw the tangent and find its gradient.

Worked example 15

The graph shows how the gravitational potential V varies with distance r from the centre of a planet of radius $2.0 \times 10^6 \text{ m}$. (a) Find the work needed to move a 2000 kg probe from the surface to $r = 8.0 \times 10^6 \text{ m}$. (b) Find g at the surface from the graph.



V-r graph for worked example 15.

1. (a) Read the graph: $V(\text{surface}) = -5.0 \times 10^7 \text{ J kg}^{-1}$, $V(8.0 \times 10^6 \text{ m}) = -1.25 \times 10^7 \text{ J kg}^{-1}$.
2. $W = m\Delta V = 2000 \times (-1.25 \times 10^7 + 5.0 \times 10^7) = 7.5 \times 10^9 \text{ J}$.
3. (b) Draw the tangent at $r = 2.0 \times 10^6 \text{ m}$. Its gradient is 25 J kg^{-1} per m (it rises about $5.0 \times 10^7 \text{ J kg}^{-1}$ over $2.0 \times 10^6 \text{ m}$).
4. $g = -\text{gradient in direction}$, size 25 N kg^{-1} , pointing towards the planet.

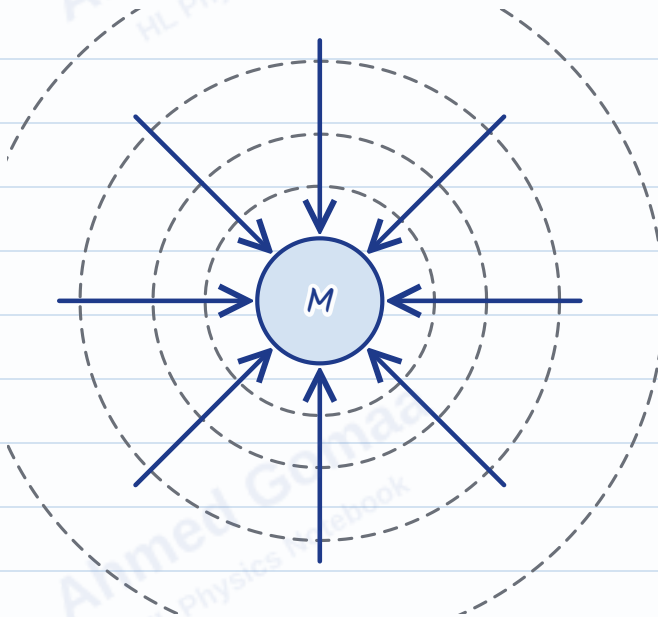
Worked example 16

On the $V-r$ graph between a planet and its moon, the potential at the planet's surface is $-6.0 \times 10^7 \text{ J kg}^{-1}$ and at the top of the hump it is $-0.5 \times 10^7 \text{ J kg}^{-1}$. Find the minimum energy a 900 kg probe needs to reach the moon from the planet's surface.

1. The probe only needs to reach the top of the hump (the zero-field point). After that the moon pulls it in.
2. $E = m\Delta V = 900 \times (-0.5 \times 10^7 - (-6.0 \times 10^7)) = 4.95 \times 10^9 \text{ J}$.

Equipotential surfaces (HL)

An equipotential surface joins points with the **same potential**. Around a sphere they are spheres (circles on paper).



dashed circles: equipotentials

arrows: field lines

field lines cross equipotentials at 90°

equal steps of V get further apart as the field weakens

moving along one: $W = 0$

Equipotentials and field lines around a sphere.

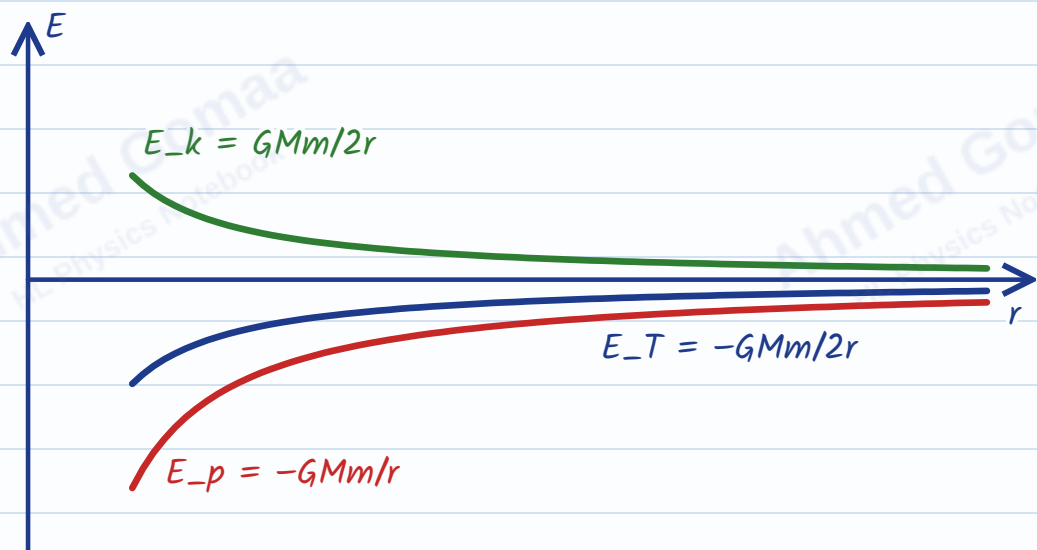
1. Field lines cross equipotentials at 90° .
2. Moving along an equipotential, $\Delta V = 0$, so **no work is done**.
3. Drawn with equal steps of V , the surfaces are closer together where the field is stronger.

Energy of an orbit (HL)

For a satellite of mass m in a circular orbit of radius r : from $v^2 = GM/r$, the kinetic energy is $E_k = \frac{1}{2}mv^2 = GMm/2r$. Add the potential energy:

$$E_k = GMm/2r \quad E_p = -GMm/r \quad E_T = E_k + E_p = -GMm/2r$$

$E_p = -2E_k$ and $E_T = -E_k$. A bound orbit always has **negative** total energy.



The three energies against orbit radius.

Worked example 17

An 800 kg satellite moves from a circular orbit of radius 7.0×10^6 m to one of radius 8.0×10^6 m. Find its total energy in the first orbit and the energy the engines must supply.

1. $E_T = -GMm/2r = -6.67 \times 10^{-11} \times 6.0 \times 10^{24} \times 800 / (2 \times 7.0 \times 10^6)$
 $= -2.3 \times 10^9$ J.

2. New orbit: $E_T = -2.0 \times 10^9$ J. Energy supplied $= \Delta E_T = +2.9 \times 10^9$

J. The satellite ends up slower (E_k falls) but with more potential energy.

Worked example 18

A probe of mass m is launched from the surface of a planet (mass M , radius R) with kinetic energy $5GMm/6R$. It later settles into a circular orbit with no further work done by its engines. Find the orbit radius.

1. Total energy at launch: $E_T = 5GMm/6R - GMm/R = -GMm/6R$. It is negative, so the probe cannot escape.

2. In orbit $E_T = -GMm/2r$. Energy is conserved: $-GMm/2r = -GMm/6R \rightarrow r = 3R$.

The total energy decides the path (no air, no engines after launch):

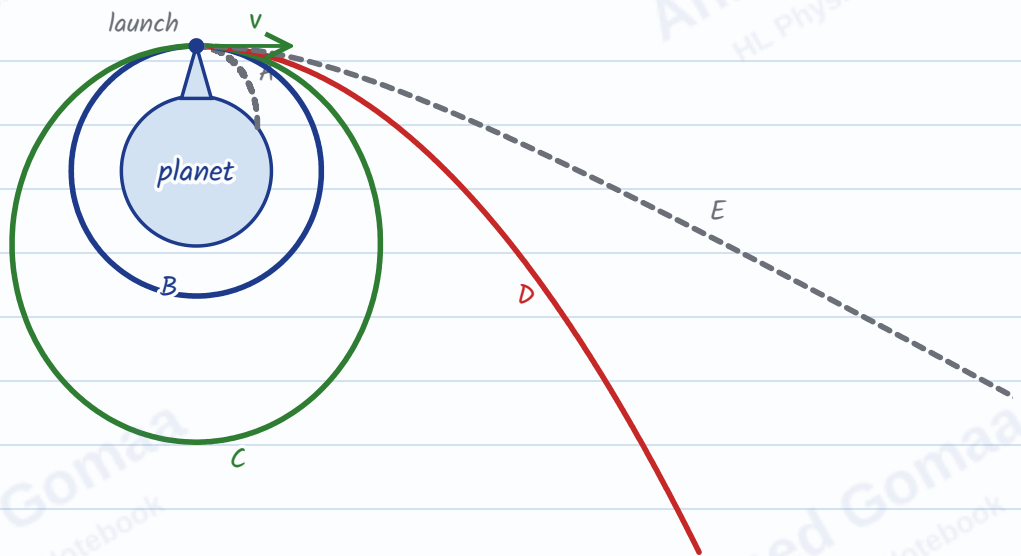
$E_T < 0$: the body is **bound** to the planet. It cannot escape: it falls back to the ground, or it moves in a **circle** or an **ellipse**. We **see it again**.

$E_T = 0$: it **just escapes** on a **parabola**, reaching infinity with zero speed. It **never returns**.

$E_T > 0$: it escapes on a **hyperbola**, still moving at infinity. It **never returns**.

Launched straight up, the path is a straight line: it falls back if $E_T < 0$ and escapes if $E_T \geq 0$.

► [See it move: Newton's cannon in the visual guide](#)



A $E_T < 0$: falls back to the ground

B $E_T < 0$: circle, $E_T = -GMm/2r$

C $E_T < 0$: ellipse, comes back

Launched straight up: a straight line,
it falls back if $E_T < 0$, escapes if $E_T \geq 0$.

D $E_T = 0$: parabola, just escapes

E $E_T > 0$: hyperbola, escapes

bound ($E_T < 0$): we see it again

free ($E_T \geq 0$): never returns

Fired sideways from the same mountain with more and more speed: the sign of the total energy decides the path.

Escape speed (HL)

Escape speed · learn it word for word

the minimum speed a body must have to reach infinity (to escape the gravitational field of the planet), with no further energy supplied.

$v_{\text{esc}} = \sqrt{2GM/r}$ It does not depend on the mass of the body.

Show that (learn this derivation)

Show that the escape speed from a point a distance r from the centre of a planet of mass M is $v_{\text{esc}} = \sqrt{2GM/r}$.

1. At infinity the body has $E_p = 0$ and, at the minimum launch speed, $E_k = 0$; its total energy there is **zero**.
2. Energy is conserved, so the total energy at launch is also zero:
 $\frac{1}{2}mv^2 - GMm/r = 0$.
3. $v_{\text{esc}} = \sqrt{2GM/r}$. In terms of the potential: $v = \sqrt{-2V}$.

× Mistake

Equating forces ($GMm/r^2 = mv^2/r$) and then putting in a factor of 2.

That is the orbit equation, not escape: escape is an **energy** argument, and the markscheme gives no marks for the force method.

Worked example 19

Find the escape speed from the Earth's surface, and the extra speed a satellite in a 300 km orbit needs to escape.

1. Surface: $v = \sqrt{2GM/R} = \sqrt{(2 \times 6.67 \times 10^{-11} \times 6.0 \times 10^{24}) / 6.4 \times 10^6}$
 $= 1.1 \times 10^4 \text{ m s}^{-1}$ (11 km s⁻¹).
2. At $r = 6.7 \times 10^6 \text{ m}$: $v_{\text{esc}} = \sqrt{2GM/r} = 1.09 \times 10^4 \text{ m s}^{-1}$, and it already moves at $v_{\text{orb}} = \sqrt{GM/r} = 7.73 \times 10^3 \text{ m s}^{-1}$.
3. Extra speed = $3.2 \times 10^3 \text{ m s}^{-1}$.

Worked example 20

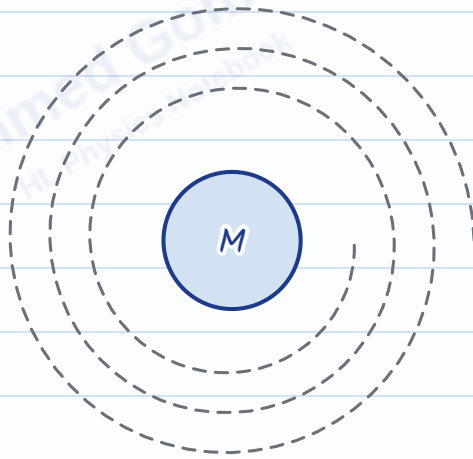
A ball is launched upwards from the surface of a planet (mass M , radius R , no air) with a speed of $v_{\text{esc}}/\sqrt{2}$. How far from the centre does it get?

1. $\frac{1}{2}mv^2 = \frac{1}{2}m \times (2GM/R)/2 = GMm/2R$. Total energy: $GMm/2R - GMm/R = -GMm/2R$.
2. At the highest point $E_k = 0$: $-GMm/r = -GMm/2R \rightarrow r = 2R$ (a height of R above the surface).

A step further: black holes

If a star is so small that its escape speed reaches the speed of light c , nothing can escape: $R = 2GM/c^2$. For a star of 10 solar masses ($2.0 \times 10^{31} \text{ kg}$) this is about $3.0 \times 10^4 \text{ m}$: the whole star squeezed into a 30 km ball.

Orbits with air resistance (HL)



air resistance takes energy:

$$E_T = -GMm/2r \text{ falls}$$

→ r gets smaller

$$v = \sqrt{GM/r} \text{ gets LARGER}$$

E_k rises, E_p falls twice as much

A small drag force makes the orbit shrink slowly.

How the marks go: "Outline the effect of a small drag force on the orbit" (3 marks)

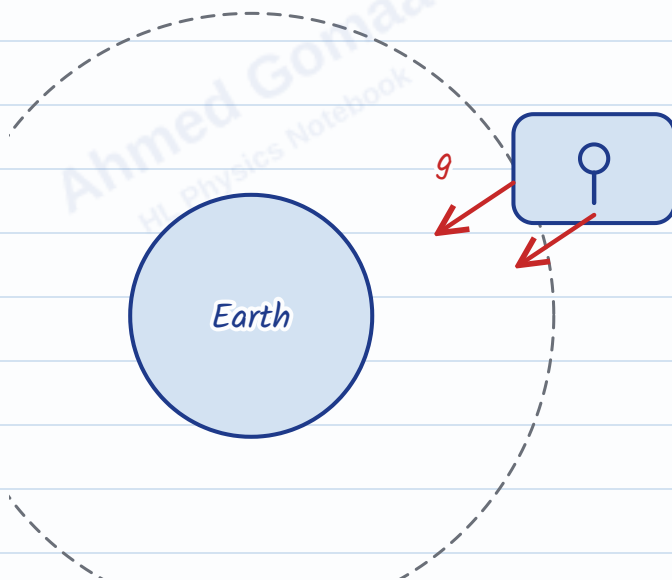
✓ The drag force does negative work: the satellite's **total energy decreases** (becomes more negative).

✓ $E_T = -GMm/2r$, so the **orbit radius decreases**.

✓ $v = \sqrt{GM/r}$, so the **speed increases** (kinetic energy increases; potential energy decreases by twice as much).

Do not write "the kinetic energy decreases": it is the total energy that falls.

Weightlessness in orbit



astronaut and craft fall
with the SAME acceleration g
so the floor pushes with $N = 0$:
they feel weightless

The astronaut and the spacecraft fall together.

At 400 km, g is still about 8.7 N kg^{-1} (worked example 4), so the astronaut is not “out of gravity”. The astronaut and the spacecraft are both in **free fall** with the **same acceleration** towards the Earth. The floor does not need to push on the astronaut, so the contact force $N = 0$ and the astronaut **feels** weightless. The weight (the gravitational force) is still there: it provides the centripetal force.

Writing the explanations

Many D.1 marks are for definitions and short derivations. Markschemes look for the same key words every time.

“Define gravitational field strength” (1–2 marks)

✓ **Force per unit mass** ✓ **acting on a small (test / point) mass** placed at that point.

Not “the acceleration due to gravity” and not “gravity”.

"Define gravitational potential at a point" (2 marks)

✓ The **work done per unit mass** ✓ in bringing a small mass **from infinity** to that point.

"Energy" is accepted for "work done". "From the point to infinity" loses the mark.

"Show that $T^2 = 4\pi^2 r^3 / GM$ " (2-3 marks)

✓ Centripetal force **equated to** the gravitational force: $GMm/r^2 = mv^2/r$.

✓ $v = 2\pi r/T$ substituted and rearranged.

Show every line: the answer is given, so the marks are for the working.

"Show that the escape speed is $\sqrt{2GM/R}$ " / "Define escape speed" (2 marks)

✓ Definition: the **minimum speed to reach infinity** / escape the planet's gravitational field. Not "to leave the orbit" or "to escape the atmosphere".

✓ Derivation: **total energy = 0** (kinetic energy gained = potential energy lost): $\frac{1}{2}mv^2 - GMm/R = 0$.

"Explain why the gravitational potential is negative" (2 marks)

✓ Potential is **zero at infinity**.

✓ The force is **attractive**, so **work must be done** (energy supplied) to move a mass to infinity.

"Outline the effect of air resistance on a satellite's orbit" (3 marks)

✓ Total energy decreases ✓ radius decreases ✓ speed increases.

"Explain why the planet moves in a circular orbit" (2 marks)

✓ A centripetal force is needed for circular motion ✓ and it is provided by the gravitational force between the planet and the star.

"Determine where the gravitational field is zero" (2 marks)

✓ Read where the gradient of the $V-r$ graph is zero (the maximum) — or — ✓ equate the two field strengths $GM/x^2 = Gm/(d - x)^2$ and solve.

Common mistakes

✗ Ten traps in D.1

1. Using the height above the surface as r . Always $r = R + h$.
2. Forgetting to square the factor for the radius in ratio problems (4 times the radius $\rightarrow$ 16 times smaller g).
3. Dropping the minus sign in $V = -GM/r$ or $E_p = -GMm/r$.
4. Using mgh for large heights. Use $-GMm/r$ at both points.
5. Deriving escape speed with forces. Escape is an energy argument: total energy = 0.
6. Defining escape speed as "the speed to leave the orbit" or "to escape the atmosphere".
7. Saying the kinetic energy of a satellite falls because of drag. The total energy falls; the speed rises.
8. Adding field strengths at the point of zero field. Set them equal; add only potentials.
9. Kepler: "equal arc lengths", "Sun at the centre", " $T \propto r^3$ ". Correct: equal areas, Sun at a focus, $T^2 \propto r^3$.
10. Writing V without a unit. Its unit is $J\ kg^{-1}$; field strength is $N\ kg^{-1}$.

D.1 and D.2 side by side

Coming next in D.2: the same structure with different letters. Swap G for $k = 1/4\pi\epsilon_0$, mass for charge, and g for E :

Gravitational (D.1)

$$F = GMm/r^2$$

$$g = F/m = GM/r^2$$

$$V_g = -GM/r$$

Electric (D.2)

$$F = kQq/r^2$$

$$E = F/q = kQ/r^2$$

$$V_e = kQ/r$$

$$E_p = -GMm/r$$

$$E_p = kQq/r$$

$$g = -\Delta V_g / \Delta r$$

$$E = -\Delta V_e / \Delta r$$

$$W = m\Delta V_g$$

$$W = q\Delta V_e$$

always attractive $\rightarrow V$ always negative

like charges repel $\rightarrow V$ has the sign of the charge

Practise next

You have read the notes. Now practise, in this order:

1. **Levelled practice**: first a ★ at Level 2 in every topic (the goal), then Levels 3 and 4 for a 6 or 7. Hints and worked answers help you as you go.
2. **IB-format practice** ([Paper 1A](#) · [Paper 2](#)): timed, one attempt, and your report goes to your teacher. Do these when the practice goal is reached.

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